Solutions to Logic Problems

1.  
(a) \( A \land (B \lor C) \) \( (A \land B) \lor C \)
Not equivalent: let \( A = F, B = F, C = T \)

(b) \( A \lor (B \land C) \) \( (A \lor B) \land C \)
This is essentially the same as part (a) (since \( \land \) and \( \lor \) are commutative operators). Not equivalent: let \( A = T, B = F, C = F \)

(c) \( A \rightarrow (B \lor C) \) \( (A \rightarrow B) \lor (A \rightarrow C) \)
Equivalent.

(d) \( A \rightarrow (B \land C) \) \( (A \rightarrow B) \land (A \rightarrow C) \)
Equivalent.

(e) \( (A \lor B) \rightarrow C \) \( (A \rightarrow C) \lor (B \rightarrow C) \)
Not equivalent: let \( A = F, B = T, C = F \).

(f) \( (A \land B) \rightarrow C \) \( (A \rightarrow C) \land (B \rightarrow C) \)
Not equivalent: let \( A = F, B = T, C = F \).

(g) \( \neg(A \rightarrow B) \) \( \neg B \rightarrow \neg A \)
Not equivalent — they give different answers for every possible value of \( A \) and \( B \). In fact, these are exact opposites, which means that \( A \rightarrow B \) is equivalent to \( \neg B \rightarrow \neg A \).
(This is the contrapositive law.)

(h) \( \neg A \rightarrow B \) \( A \rightarrow \neg B \)
Not equivalent: let \( A = F, B = F \).

2. Bring the negation symbol inside the following formula as far as you can:
\[ \neg ((\exists x)(\forall y)(\exists z)x \lor y \rightarrow z) \]
\[ (\forall x)(\exists y)(\forall z)(x \lor y) \land \neg z \]

3. Using resolution, show that the set of rules and facts given by:
\[ \neg B \lor \neg C \lor A \]
\[ \neg D \lor B \]
can be used to prove $A$.

We add the "fact" $\neg A$ to the knowledge base and then proceed to produce a contradiction.

(a) $\neg B \lor \neg C \lor A$
(b) $\neg D \lor B$
(c) $C$
(d) $D$
(e) $\neg A$

Combining (a) and (e) gives us (f): $\neg B \lor \neg C$. Combining (c) and (f) gives us (g): $\neg B$. Combining (b) and (d) gives us (h): $B$. Our contradiction is (g) and (h): $B \land \neg B$. Therefore, the thing that caused the contradiction, $\neg A$, must be false, so $A$ must be true.